**Characteristics:**

May have ***\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_***. This is a line that the function approaches more and more closely, but doesn't ever touch.

For a rational function of the form,



There is a **\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_** at the line x = **\_\_\_\_\_**, and the domain is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

*(All of the x-values, except when x = h)*

There is a **\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_** at the line y = **\_\_\_\_\_**, and the range is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

*(All of the y-values, except when y = k)*

**Example # 1: Describe the transformations and identify the asymptote(s), domain, and range for the function:**



Vertical Asymptote: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Horizontal Asymptote: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Domain: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Range: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Zeros and Asymptotes:**

If \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, where p and q are polynomial functions written in standard form with no common factors other than 1, then the function f(x) has

* zeros at each real value of x for which \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.
* a vertical asymptote at each real value of x for which \_\_\_\_\_\_\_\_\_\_.
* a horizontal asymptote based on the degree of p(x) and q(x).
* If the degree of p \_\_\_\_\_ degree of q, there is \_\_\_\_\_ horizontal asymptote.
* If the degree of p \_\_\_\_\_ degree q, the horizontal asymptote is the line \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.
* If the degree of p \_\_\_\_\_\_ degree of q, the horizontal asymptote is the line

***Some rational functions may actually \_\_\_\_\_\_\_\_\_\_\_\_\_\_ the horizontal asymptote!***

**Example # 2: Identify the zeros and asymptotes for the function below.**



***Step 1: Factor the numerator and denominator.***

***Step 2: Set each factor in the numerator equal to zero and solve to find the zeros of the function.***

***Step 3: Set each factor in the denominator equal to zero and solve to find the vertical asymptote(s).***

***Step 4: Compare the degree of p(x) to the degree of q(x) to determine the horizontal asymptote.***

**Example # 3: Identify the zeros and asymptotes of the function below.**



***Step 1: Factor the numerator and denominator.***

***Step 2: Set each factor in the numerator equal to zero and solve to find the zeros of the function.***

***Step 3: Set each factor in the denominator equal to zero and solve to find the vertical asymptote(s).***

***Step 4: Compare the degree of p(x) to the degree of q(x) to determine the horizontal asymptote.***

*In some cases, both the numerator and denominator of a rational function will equal 0 for a particular x-value. As a result, the function will be undefined at this x-value. When this happens, the graph of the function may have a \_\_\_\_\_\_\_\_\_\_\_\_. A hole is an \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ point(s) in a graph.*

**Holes in Graphs:**

If a rational function has the same factor \_\_\_\_\_\_\_\_\_\_\_\_\_\_ in both the numerator and denominator, then there is a hole in the graph at the point where \_\_\_\_\_\_\_\_\_\_\_\_\_\_, unless the line x = b is a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ asymptote.

**Example # 4: Identify the holes in the graph below.**



***Step 1: Factor the numerator and denominator.***

***Step 2: Identify common factors in the numerator and denominator.***

***Step 3: Set the common factor equal to zero and solve.***

***Step 4: Substitute the holes x-value in to the remaining factors of the function and simplify to find your y-value.***